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book, p. 12, under the form "If the hypothesis of [a] right angle is true in a single case, it holds true in every case." proved by Saccheri without any supposition on the length of the line and a century before Legendre.

Again, not even the name of Schweikart is mentioned, though as I have shown in THE AMERICAN MATHEMATICAL MONTHLY, Vol. VII., pp. 247-252, and in "Science," Vol. XII., pp. 842-846, Schweikart may be considered the first to publish a genuine conscious treatise on Non-Euclidean Geometry [which I there give for the first time in English]. This fixes the date of the first conscious creation and naming of the Non-Euclidean Geometry as between 1812 and 1816.

We may perhaps timidly hope that the great Jesuit, Saccheri, had some suspicion of what he had really done. But meanwhile it seems almost certain that he really believed that his beautiful Non-Euclidean Geometry was all a reductio-ad-absurdum, and that he had really justified the title of his book, "Euclid Vindicated from Every Fleck," by proving Non-Euclidean Geometry untenable. On the other hand we of the new school, followers of Schweikart and Bolgai János, believe that it is our Non-Euclidean Geometry itself which finally vindicates Euclid from every fleck, and justifies the weighty tribute of Professor Alfred Baker: "Of the perfection of Euclid (B. C. 290) as a scientific treatise, of the marvel that such a work could have been produced two thousand years ago, I shall not here delay to speak. I content myself with making the claim that, as a historical study, Euclid is, perhaps, the most valuable of those that are taken up in our educational institutions."

Austin, Texas.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

115. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah Iowa.

Find the conditions of the coefficients of a general biquadratic equation so that it may be solved by quadratics.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and the PROPOSER.

To solve as a pure quadratic, without using the cubic, we might proceed as follows:

Let $x^4+ax^3+bx^2+cx+d=0$ be the general biquadratic.

(1) Then $(x^2+\frac{1}{2}ax)^2+bx^2-\frac{1}{4}a^2x^2+cx+d=0$.

Now if $c=\frac{1}{2}ab-\frac{1}{8}a^3$, we get $(x^2+\frac{1}{2}ax)^2+(b-\frac{1}{4}a^2)(x^2+\frac{1}{2}ax)+d=0$.

This equation can be solved by quadratics.

(2) Let $d=c^2/a^2$. Then $x^4+ax^3+bx^2+cx+c^2/a^2=0$.

$\therefore (x^2+\frac{p}{2}x+\frac{c}{a})^2=(\frac{p^2}{4}+\frac{2c}{a}-b)x^2$.

This is easily solved by quadratics.

This gives two conditions : 1st, $c=\frac{1}{2}ab-\frac{1}{8}a^3$; $d=c^2/a^2$.

II. Solution by HARRY S. VANDIVER, Bala, Pa.

The problem, I take it, is equivalent to the following:

What is the condition that the general biquadratic equation has a root which may be expressed in quadratic surds?

(A) Suppose the equation to be reduced to the form

$$x^4+qx^2+rx+s=0\ldots.(1),$$

where q, r, and s are rational; and assume that this is equivalent to

$$(x^2-kx+m)(x^2+kx+l)=0\ldots.(2);$$

then, following Descartes' method, we find as the cubic resolvent in k^2,

$$k^6+2qk^4+(q^2-4s)k^2-r^2=0\ldots.(3).$$

(*Hall and Knight's Higher Algebra*, page 485).

Now since x is to be found in terms of k, l, and m from (2), and, if it is to be in the form of a quadratic surd, then (3) must have a root in the form of a quadratic surd. Therefore, the roots of (3) must be of the forms $\pm\sqrt{b}$, and $\pm\sqrt{(c\pm\sqrt{d})}$, where b, c, and d are rational. It is sufficient to consider one value of k; put $k=\sqrt{b}$, and substituting in (2), we obtain results of the form

$$x=\begin{cases} \sqrt{m}+\sqrt{(g+n\sqrt{m})}\ldots.(4), \\ \sqrt{m}-\sqrt{(g+n\sqrt{m})}\ldots.(5), \\ -\sqrt{m}+\sqrt{(g-n\sqrt{m})}\ldots.(6), \\ -\sqrt{m}-\sqrt{(g-n\sqrt{m})}\ldots.(7), \end{cases}$$

where m, g, and n are rational.

These values may be taken as the general forms of quadratic surd roots of biquadratics. Hence the *condition that the general biquadratic has roots of the form* (4), (5), (6), and (7) *is that* (3) *has a root in the form* $\sqrt{b}$, *where* b *is rational.*

(B) *Special forms of biquadratics. Relations existing between the coefficients.*

Let the general biquadratic be

$$x^4+px^3+qx^2+rx+s=0\ldots.(9),$$

and assume that it can be thrown into the form

$$(ax^2+bx)^2+c(ax^2+bx)+d=0\ldots(10);$$

then, expanding, and removing the term involving x^3 (by the proper linear transformation) the coefficient of x also vanishes, and from this property we find that the relation between the coefficients of (9) is :

$$p^3-4pq+8r=0,$$

and in (10), the sum of two of the roots is equal to the sum of the other two.

(Cf. Burnside and Panton's *Theory of Equations*, page 41).

2nd Case. Assume that (9) can be put in the form

$$(mx+\frac{n}{x})^2+g(mx+\frac{n}{x})+f=0\ldots.(11).$$

Multiplying out, and comparing with (9), we find that the relation between the coefficients is $r^2=p^2s$, *and the roots of* (11) *are in geometrical progression.*

Also solved by *CHARLES C. CROSS*, *B. L. REMICK*, *J. SCHEFFER*, *H. C. WHITAKER*, and *J. W. YOUNG.*

116. Proposed by **ARTEMAS MARTIN, A. M., Ph. D., LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Solve the equations :

$$w(xy+yz+xz)=a\ ;\ x(wy+wz+yz)=b\ ;$$

$$y(wx+wz+xz)=c\ ;\ z(wx+wy+xy)=d.$$

I. Solution by **G. B. M. ZEZR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; J. M. HOWIE, The Nebraska State Normal School, Peru, Neb.; B. L. REMICK, Bradley Polytechnic Institute, Peoria, Ill.; NOAH ADAIR, A. M., Tyler College, Tyler, Tex.; G. I. HOPKINS, High School, Manchester, N.H.; FREMONT CRANE, B. S., C. E., Stockett, Mont.; H. C. WHITAKER, Ph.D., Manual Training School, Philadelphia, Pa., C. C. CROSS, Meredithville, Va.; L. B. FILLMAN, Chester, Pa.; O. S. WESTCOTT, A. M., Sc. D., North Division High School, Chicago, Ill.; and W. W. LANDIS, A. M., Dickinson College, Carlisle, Pa.**

(1)+(2)+(3)+(4) gives $wxy+wxz+wyz+xyz=\frac{1}{3}(a+b+c+d)\ldots.(5)$.

(5)−(1) gives $xyz=\frac{1}{3}(b+c+d-2a)\ldots.(6)$.

(5)−(2) gives $wyz=\frac{1}{3}(a+c+d-2b)\ldots.(7)$.

(5)−(3) gives $wxz=\frac{1}{3}(a+b+d-2c)\ldots.(8)$.

(5)−(4) gives $wxy=\frac{1}{3}(a+b+c-2\mathrm{d})\ldots.(9)$.

(6)÷(7) gives $x/w=(b+c+d-2a)/(a+c+d-2b)$.

(6)÷(8) gives $y/w=(b+c+d-2a)/(a+b+d-2c)$.

(6)÷(9) gives $z/w=(b+c+d-2a)/(a+b+c-2d)$.

These values of x, y, and z in (1) give

$$w=\sqrt[3]{\frac{(a+b+c-2d)(a+b+d-2c)(a+c+d-2b)}{3(b+c+d-2a)^2}}=\sqrt[3]{\frac{BCD}{3A^2}},\text{ suppose.}$$

Similarly, $x=\sqrt[3]{}(ACD/3B^2)$, $y=\sqrt[3]{}(ABD/3C^2)$, $z=\sqrt[3]{}(ABC/3D^2)$.